Acoustoelectric Effects with Hypersonic Waves of Large Amplitude

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The classical treatment of acoustoelectric interactions in piezoelectric semiconductors is extended into the range where the amplitude of the potential wave ϕ_0 is not small compared with kT/e. It is assumed that the wavelength is short compared with the Debye length so that the effect of the electronic space charge may be neglected. It is shown that when $e\phi_0 > 2kT$ an electric field in the direction of sound propagation may cause an instability in the electron current owing to the change in electron temperature with field. The conditions for such an instability are derived in terms of the electron energy relaxation time τ and an effective diffusion time for the electrons $\lambda^2/4\pi D$.

1. INTRODUCTION

THE "acoustoelectric effect" was first discussed by Parmenter[1] in 1953. Since then there have been a great many papers on this and allied effects. Weinreich[2] considered acoustoelectric interactions semiclassically and showed that acoustic amplification should be possible in the presence of a suitable applied electric field. In order to simplify his treatment he assumed charge neutrality, although this is not a necessary condition for a substantial acoustoelectric effect if the acoustic wavelength (λ) is short enough. This is evident from the work of Weinreich *et al.*[3] who also give the relation between the acoustoelectric current and the acoustic attenuation. This provides a link with the theoretical and experimental results of Hutson and White[4–6] on acoustic attenuation and amplification, and Wang[7] has measured an acoustoelectric current in CdS at 33 Mc/sec in fair agreement with theory.

These classical theories are appropriate if the electron mean free path $(l) \ll \lambda$. The case of $l > \lambda$ has been considered by a number of workers, and Spector[8] and Pippard[9] have shown that the general features of the classical results are also found from a quantum treatment.

The purpose of the present paper is to consider acoustoelectric effects when the sound has a large amplitude. The treatment is semiclassical and similar to that of Weinreich except that the restriction of $e\phi_0/kT \ll 1$ is removed. It is also assumed that the wavelength is short enough, and the carrier concentration low enough, that the space charge of the bunched carriers (electrons) may be neglected. This second condition corresponds to

$$(\lambda/2\pi)^2 \ll L_D^2, \tag{1}$$

where L_D = Debye length, or

$$\omega_C \omega_D \ll \omega^2$$

[Hutson and White (Ref. 4) terminology].

It is shown that when $e\phi_0 \gg kT$ the electrons are effectively trapped [their drift velocity (v_d) is nearly equal to that of the sound (v_s)] providing the electron mobility (μ) is so high that the field required to drive them with the velocity of sound (v_s/μ) is much less than the maximum field in the wave ($2\pi\phi_0/\lambda$). It is also shown that when $e\phi_0 > 2kT$ the current density (j) versus field (E) characteristic may show an instability over a certain range of E. Physically this corresponds to an unstable avalanche of electrons escaping from the troughs of the potential wave owing to the applied field. The escaping electrons are heated by the field and, by sharing their energy with other trapped electrons, cause further electrons to escape. The process may therefore be regenerative.

The conditions for instability are discussed in section 4 in terms of the analysis given in Secs. 3 and 4.

2. LIST OF SYMBOLS

A,B	Constants
D	Electron diffusion constant
e	Electronic charge
ϵ_e	Average electron energy
E	Electric field in the direction of the sound
ϕ	Potential due to the wave, amplitude ϕ_0
l	Electron mean free path
λ	Sound wavelength
L_D	Debye length $(\epsilon kT/e^2 n_0)^{1/2}$
μ	Electron mobility
n	Electron concentration
n_0	Equilibrium value of n
t_0	effective electron transit time [Eqs. (28), (28a)]
τ	electron energy relaxation time [Eq. (25)]
T	electron temperature
T_L	lattice temperature
T_0	modified lattice temperature, defined by Eq. (26)

* Much of this work was performed during a visit to the IBM Watson Research Center, Yorktown Heights, New York.

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[2] G. Weinreich, Phys. Rev. **104**, 321 (1956).
[3] G. Weinreich, T. M. Saunders, Jr., and H. G. White, Phys. Rev. **114**, 33 (1959).
[4] A. R. Hutson and D. L. White, J. Appl. Phys. **33**, 40 (1962).
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[7] W-C Wang, Phys. Rev. Letters **9**, 443 (1962).
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[9] A. B. Pippard, Phil. Mag. **8**, 161 (1963).

u_0 $e\phi_0/kT_0$
v_d electron drift velocity
v_s velocity of the sound wave
y normalised electric field [Eqs. (28), (28a)]
z normalised electron drift velocity [Eqs. (28), (28a)].

3. ANALYSIS

3.1 Derivation of the Basic Equation for j in Terms of E and ϕ

Assuming the electron distribution is Maxwellian, the one-dimensional continuity equation for electrons may be written

$$D\left[\frac{\partial^2 n}{\partial x^2}-\frac{e}{kT}\frac{\partial}{\partial x}\left(n\frac{\partial\psi}{\partial x}\right)\right]-\frac{\partial n}{\partial t}=0, \tag{2}$$

where D is the electron diffusion constant, n is the electron concentration, and ψ is the chemical potential of the electrons which we shall take equal to the electrostatic potential. Equation (2) may be solved by assuming a steady state in a frame of reference traveling with the velocity of sound.

Let

$$\begin{aligned}\partial\phi/dx&=\partial\psi/dx-E,\\ x'&=x-v_st,\\ v'_s&=v_s-\mu E,\end{aligned} \tag{3}$$

where μ is the electron mobility. It may be seen that ϕ, x', and v_s' correspond, respectively, to the potential due to the sound wave, the distance in a frame of reference traveling with the velocity of sound, and the electron drift velocity relative to that of sound.

The continuity equation now becomes

$$D\left[\frac{d^2n}{dx'^2}-\frac{e}{kT}\frac{d}{dx'}\left(n\frac{d\phi}{dx'}\right)\right]+v_s'\frac{dn}{dx'}=0, \tag{4}$$

which by integration gives

$$D\left[\frac{dn}{dx'}-\frac{e}{kT}n\frac{d\phi}{dx'}\right]+v_s'n=A, \tag{5}$$

where A is the electron flux in the moving frame of reference which is related to the true electron drift velocity (v_d) by the relation

$$A=n_0(v_s-v_d), \tag{6}$$

where n_0 is the average electron concentration.

Equation (5) may be solved by letting

$$n=g(x')\exp[e\phi(x')/kT], \tag{7}$$

which leads to

$$\frac{d[g(x')]}{dx'}+\frac{v_s'}{D}g(x')=\frac{A}{D}\exp\left[\frac{-e\phi(x')}{kT}\right] \tag{8}$$

and by letting

$$g(x')=\exp\left[-\left(\frac{v_s'}{D}\right)x'\right]\times\left\{B+\frac{A}{D}\int_0^{x'}\exp\left[-\frac{e\phi}{kT}+\left(\frac{v_s'}{D}\right)x'\right]dx'\right\}, \tag{9}$$

where B is a constant that may be found from the periodicity condition that $g(0)=g(\lambda)$.

$$\therefore\quad B=\frac{A}{D}\left[\exp\left(\frac{v_s'}{D}\right)\lambda-1\right]^{-1}\times\int_0^{\lambda}\exp\left(-\frac{e\phi}{kT}+\frac{v_s'}{D}x'\right)dx'. \tag{10}$$

Substitution of (10) in (9) and (9) and (7) gives

$$n=\frac{A}{D}\exp\left(\frac{e\phi}{kT}-\frac{v_s'}{D}x'\right)\left[\exp\left(\frac{v_s'}{D}\lambda\right)-1\right]^{-1}\left[\int_0^{\lambda}\exp\left(-\frac{e\phi}{kT}+\frac{v_s'}{D}x'\right)dx'\right][1+f(\phi,x')], \tag{11}$$

where

$$f(\phi,x')=\left[\exp\left(\frac{v_s'}{D}\lambda\right)-1\right]\int_0^{x'}\exp\left(-\frac{e\phi}{kT}+\frac{v_s'}{D}x'\right)dx'\Big/\int_0^{\lambda}\exp\left(-\frac{e\phi}{kT}+\frac{v_s'}{D}x'\right)dx'. \tag{12}$$

v_d may now be found from Eqs. (6) and (11) using the relation

$$n_0=\frac{1}{\lambda}\int_0^{\lambda}ndx'. \tag{13}$$

$$\therefore\quad v_d=v_s-D\lambda\left[\exp\left(\frac{v_s'}{D}\lambda\right)-1\right]\Big/\int_0^{\lambda}\exp\left(-\frac{e\phi}{kT}+\frac{v_s'}{D}x'\right)dx'\int_0^{\lambda}\exp\left(\frac{e\phi}{kT}-\frac{v_s'}{D}x'\right)[1+f(\phi,x')]dx'. \tag{14}$$

$f(\phi,x')$ is given by Eq. (12).

3.2. Solution of Eq. (14) in the Limit that $|v_s'\lambda/D|\ll 1$

This condition means physically that the electrons are so mobile that their spacial distribution in the moving frame of reference is independent of v_s.

Equation (14) then becomes

$$v_d=v_s-v_s'\lambda^2\bigg/\int_0^\lambda \exp\left(-\frac{e\phi}{kT}\right)dx'\int_0^\lambda \exp\left(+\frac{e\phi}{kT}\right)dx'. \quad (15)$$

For a sinusoidal sound wave

$$\phi=\phi_0\cos(2\pi x'/\lambda). \quad (16)$$

$$\therefore v_d=v_s-\{v_s'/[I_0(e\phi_0/kT)]^2\}, \quad (17)$$

where I_0 is the zero-order Bessel function of imaginary argument. This equation is plotted in Fig. 1 and shows that the electrons are effectively trapped when $e\phi_0\gtrsim 2kT$.

3.3. Approximate Solution of Eq. (14) in the Limit that $e\phi_0\gg kT$

An alternative approximation for the solution of Eqs. (14) and (16) is to assume that

$$\int_0^\lambda \exp\left[-\frac{e\phi_0}{kT}\cos\left(\frac{2\pi x'}{\lambda}\right)+\frac{v_s'}{D}x'\right]dx' \quad (18)$$

can be found by considering only the regions in which $\cos(2\pi x'/\lambda)$ is nearly equal to -1. This approximation is valid if

$$e\phi_0/kT\gg 1 \quad (19)$$

and

$$(\lambda v_s'/D)^2\ll 4\pi e\phi_0/kT. \quad (20)$$

In these limits it is possible to evaluate the expression (18) by treating $v_s'x'/D$ as a constant with a value found from the value of x' at which

$$\cos(2\pi x'/\lambda)=-1.$$

A similar procedure may be used to find the second integral in Eq. (14) although it is necessary to divide the integral into two; an integration from 0 to $\lambda/2$ and another from $\lambda/2$ to λ. The effective value of $f(\phi,x')$ is zero between 0 and $\lambda/2$ and $\exp(v_s'\lambda/D)-1$ between $\lambda/2$ and λ.

Using these approximations

$$v_d=v_s-\frac{D\lambda[\exp(\lambda v_s'/2D)-\exp(-\lambda v_s'/2D)]}{4\left[\int_0^{\lambda/2}\exp(e\phi/kT)dx'\right]^2}. \quad (21)$$

Since the integral in the denominator is significant only when $\cos(2\pi x'/\lambda)\sim 1$ it may be evaluated by taking the first-order expansion of $\cos(2\pi x'/\lambda)$ and integrating form 0 to ∞.

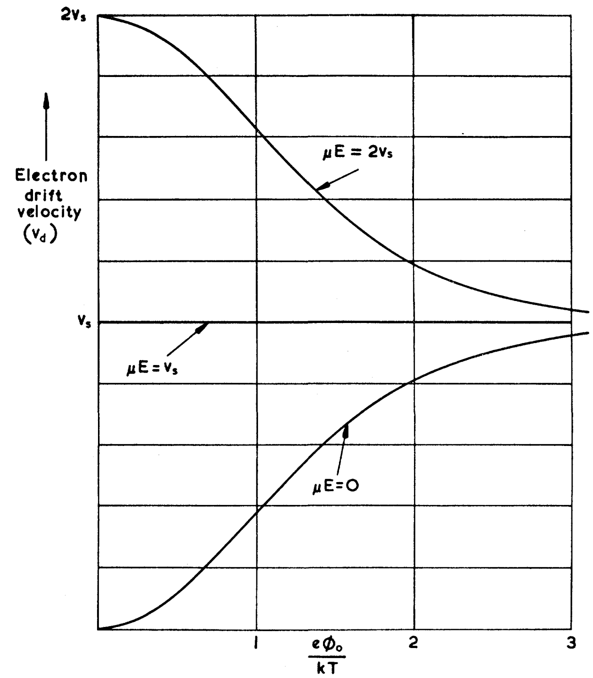


FIG. 1. Electron drift velocity versus amplitude of potential wave, from Eq. (17).

$$\therefore v_d=v_s-(4\pi\mu\phi_0/\lambda)\sinh(\lambda v_s'/2D)\times\exp(-2e\phi_0/kT). \quad (22)$$

It may be shown that Eq. (22) predicts nearly the same value of v_d as Eq. (17) if $e\phi_0/kT>\sim 2$ [as required for the derivation of (22)] and $\lambda v_s'/D\ll 1$ [as required for Eq. (17)].

4. INSTABILITY IN THE ELECTRON CURRENT

Equation (22) may be used to predict the conditions for an instability of the electron current (en_0v_d). The instability arises in a region where $|\lambda v_s'/2D|>1$. The accuracy of Eq. (22) is reduced in this range owing to the derivation being dependent on the inequality 20, this should not introduce a serious error in the range of practical interest.

Supposing that the electric field is in the direction that aids the drag of the electrons by the sound ($E>0$) and taking the limit $-\lambda v_s'/2D>1$, Eq. (22) may be written

$$v_d-v_s=(2\pi\mu\phi_0/\lambda)\exp(-\lambda v_s/2D)\times\exp[-e(2\phi_0-\lambda E/2)/kT]. \quad (23)$$

It is interesting that the second exponential term has a clear physical interpretation in that it corresponds to the height of the barrier between one trough and the next (Fig. 2).

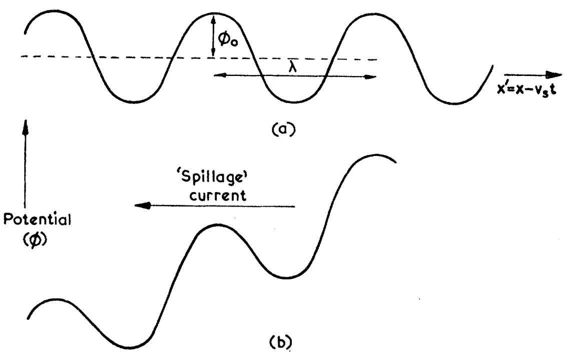


FIG. 2. Electric field opposing the drag of electrons by the sound. (a) No field, (b) with opposing field.

The temperature T in all the above equations may differ from the lattice temperature T_L. The electron distribution is being heated by interaction with the sound wave and by the electric field.[10]
It is difficult to obtain an exact expression for the rate of energy input per electron ($d\epsilon_e/dt$), but the equation

$$d\epsilon_e/dt = ev_s(v_s/\mu - \mathrm{E}) + ev_dE \tag{24}$$

should be a fairly good approximation in the range where Eq. (22) applies. The first term is due to the sound, and gives the correct value when $E = v_s/\mu$ and a good approximation when $E = 0$. The linear dependence on field is found from the small signal theory of acoustic attenuation[4–6] in the range defined by inequalities (1) and (20) and should be a reasonable approximation in the large signal case in the range where Eq. (22) applies. The second term arises from the principle of superposition.

If one assumes that the electron energy relaxation processes can be characterized by a single time constant τ

$$k(T - T_L)/\tau = d\epsilon_e/dt = eE(v_d - v_s) + ev_s^2/\mu \tag{25}$$

writing

$$T_0 = T_L + ev_s^2\tau/k\mu \tag{26}$$

and substituting (25) in (23)

$$v_d - v_s = \frac{2\pi\mu\phi_0}{\lambda}\exp(-\lambda v_s/2D) \times \exp\frac{-e(2\phi_0 - \lambda E/2)}{kT_0[1 + e\tau(v_d - v_s)E/kT_0]}. \tag{27}$$

This equation may be simplified by defining the following parameters.

$$z = (v_d - v_s)(\lambda/2\pi\mu\phi_0)\exp(+\lambda v_s/2D) \quad \text{(A normalized velocity)}$$
$$y = E\lambda/2\phi_0 \quad \text{(A normalized field)} \tag{28}$$
$$u_0 = e\phi_0/kT_0 \quad \text{(A normalized amplitude)}$$
$$t_0 = (\lambda^2/4\pi D)\exp(\lambda v_s/2D) \quad \text{(An effective transit time).}$$

Thus (27) becomes

$$z = \exp\left[\frac{-u_0(2 - y)}{1 + (u_0^2\tau/t_0)zy}\right]. \tag{29}$$

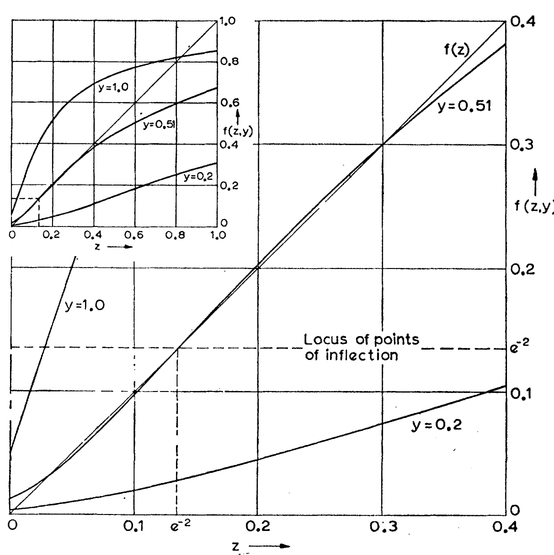
If the field opposes the sound, exactly the same equation is obtained. However, in this case the parameters are redefined thus,

$$z = -(v_d - v_s)(\lambda/2\pi\mu\phi_0)\exp(-\lambda v_s/2D), \tag{28a}$$
$$y = -E\lambda/2\phi_0,$$
$$t_0 = (\lambda^2/4\pi D)\exp(-\lambda v_s/2D).$$

The characteristics of Eq. (29) may be understood with the aid of an analysis based on a graph of the left-hand side [$f(z)$] and the right-hand side [$f(z,y)$] of the equation (see Fig. 3). The condition that Eq. (29) is satisfied for more than one value of z is that the lines $f(z)$ and $f(z,y)$ intersect more than once.

$$\text{Since } f(z) > f(z,y) \quad \text{when } z \to \infty$$
$$\text{and } f(z) < f(z,y) \quad \text{when } z \to 0,$$

it is only possible for z to be multivalued if $f(z,y)$ has a point of inflection. By differentiating $f(z,y)$ twice with respect to z and equating to zero, the condition for



Normalized electron drift velocity [Eqs. (28), (28a)]

FIG. 3. The right-hand side of Eq. (29) [$f(z,y)$] plotted for $u_0 = 3$ and $\tau/t_0 = 2$. The condition for instability is found from the slope of $f(z,y)$ at $f(z,y) = e^{-2}$ (the point of inflection) when y is adjusted to make $z = e^{-2}$ at this point. In this case the critical value of y is 0.51 and an instability is shown by the three values of z that satisfy Eq. (29).

[10] V. B. Sandomirskii and Sh. M. Krogan, Fiz. Tverd. Tela 5, 1894 (1963) [English transl.: Soviet Phys.—Solid State 5, 1383 (1964)].

this is found to be that

$$f(z,y)=e^{-2}. \tag{30}$$

The condition for z to be multivalued is that when y is adjusted to the critical value (y_c) that makes $f(z,y)=f(z)$ at the point of inflection $[f(z,y)=e^{-2}]$, the slope of $f(z,y)$ should be greater than unity [the slope of $f(z)$]. This is a necessary as well as a sufficient condition. If the curve of $f(z,y)$ for $y=y_c$ only intersects the line $f(z)$ at one point, no other value of y can produce more than one intersection as this would require an intersection with the curve for $y=y_c$. This is impossible because $d[f(z,y)]/dy$ is always positive.

Thus it is found that for an instability

$$u_0>1+(1+e^2t_0/2\tau)^{1/2}. \tag{31}$$

Equation (31) shows that the minimum value of $u_0(u_{\min})$ is 2 if $t_0/\tau\rightarrow 0$ but that $u_{\min}$ rises as t_0/τ increases. When $t_0/\tau=1$, $u_{\min}$ is about 3.2.

Equations (28) and (28a) show that if the field is opposing the sound the value of t_0 is less than with a field aiding the sound. Thus, from Eq. (31), an instability is more readily obtained with an opposing field. This is plausible because, as may be seen from Fig. 4, when the field is opposing the sound, there is a large discrepancy between the current when the electrons are cold and "trapped" and the current when they are substantially heated by the field.

5. DISCUSSION

In order to clarify the physical requirements for the instability, it is worth restating and discussing the assumptions and conclusions from Sec. 4. An instability in the j-E characteristic should be observed if all the following conditions are satisfied.

(a) From Eqs. (31) and (28), $e\phi_0>\sim 3kT_0$.

(b) The electron concentration is large enough to allow energy sharing between the electrons. This requirement is implicit in the formulation of a single energy relaxation time τ[Eq. (25)] and the conditions for this are discussed by Frölich and Paranjape[11] and Stratton.[12]

(c) From Eq. (31), τ must be of the order of, or greater than, the effective transit time (t_0) of the electrons diffusing from one trough to the next $\{t_0\sim\lambda^2/4\pi D$, but depends on the direction of the field [Eqs. (28), (28a)]$\}$. If τ is large μ must also be large in order that T_0 should not greatly exceed T_L [Eq. (26)]. If $T_0\gg T_L$ it may be impossible to satisfy condition (a).

(d) λ must be small in order to satisfy condition (c) and also to satisfy inequality (1) with the electron concentration required for condition (b).

[11] H. Frölich and B. V. Paranjape, Proc. Phys. Soc. (London) **B69**, 21 (1956).

[12] R. Stratton, Proc. Roy. Soc. (London) **A246**, 406 (1958).

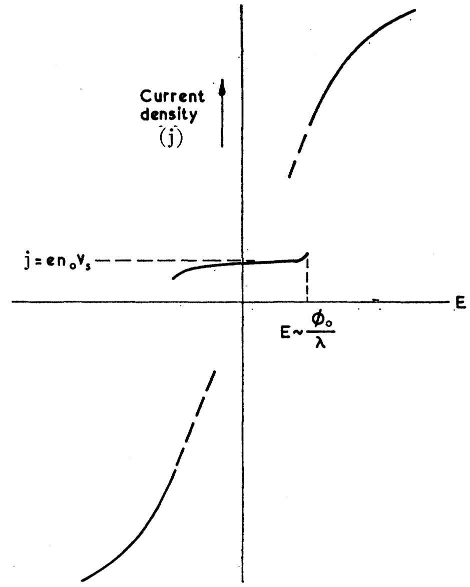


FIG. 4. Current density versus field in the direction of the sound [$e\phi_0\gg kT$, $(\lambda v_s/D)^2\ll 4\pi e\phi_0/kT$, $L_D^2\gg(\lambda/2\pi)^2$, $\tau\gtrsim t_0$].

(e) D must be large to satisfy inequality (20) and condition (c).

A number of complicating factors have been neglected in the interests of simplicity. Amongst these may be listed:

(1) The variation of μ and D with electron temperature. It may be seen from Eq. (23) that if μ increases with temperature (ionized impurity scattering) an instability may be produced with a smaller ϕ_0.

(2) The variation of ϕ_0 with distance due to attenuation or amplification of the sound by the electrons. For rather low-electron concentrations, as required by inequality (1), this should not be a very serious complication especially as the attenuation constant diminishes when $e\phi_0>kT$. This may be seen from Fig. 1 which shows that the power absorbed by the electrons saturates for large $e\phi_0/kT$.

(3) The use of a classical treatment which requires $l\ll\lambda$ in the range where λ must be very small [condition (d)]. This limits the range of conditions in which the theory is valid quantitatively. In order that the range may be reasonably wide it is necessary that the momentum relaxation time (which is proportional to l) be much less than τ. This condition should be satisfied if ionized impurity scattering is dominant. However, the high values of D and μ that are required [conditions (c) and (e)] make the condition $l\ll\lambda$ difficult to satisfy fully. In practice, therefore, the theory given will generally be only approximately valid.

It is interesting to note that the negative resistance mechanism is in some ways analogous to the mechanism proposed by Yamashita[13] to explain the negative resistance observed in impact ionisation of impurities in semiconductors. During impact ionization there are also "trapped" and "free" electrons and providing the trapped electrons can contribute significantly to the energy relaxation of the whole electron system, a negative resistance may be expected. In the present theory the trapped electrons contribute to the electron energy relaxation via electron-electron collisions.

The frequency response of the negative resistance associated with the instability depends on τ or on the dielectric relaxation time (t_D), whichever is longer. The latter may be related to the transit time t_0 by writing inequality (1) as

$$\lambda^2 = \delta L_D{}^2, \quad \text{where } \delta \ll 1.$$

Since

$$t_0 \sim \lambda^2/4\pi D \text{ [Eqs. (28), (28a)]},$$

$$t_D \sim t_0 \cdot 4\pi/\delta.$$

Since τ must be of the order of t_0, we may regard the frequency response as being largely determined by the transit time t_0. This is short if λ^2/D is small, which condition is also required for the negative resistance itself, so a high-speed process should be obtainable.

[13] J. J. Yamashita, J. Phys. Soc. Japan **17**, 884 (1962).

The experimental conditions required for observing the effects predicted are not easy to specify owing to the interdependence of the vital parameters. However, the material used must have a reasonable piezoelectric constant, a high-electron mobility and a low-electron concentration (for example, good-quality semi-insulating GaAs). Lowering the lattice temperature reduces the strain required, decreases the minimum permissible electron concentration and reduces lattice attenuation of the hypersonic wave, but the temperature must not be too low because D must be adequate [inequality (20)]. Using GaAs and a temperature of about 20°K a strain of $>\sim 8\times 10^{-5}$ would be required ($e\phi_0/kT_L > 3$). With a 10-kMc/sec wave, an electron concentration between about 10^{11} and 10^{13} cm^{-3} and an electron mobility $>\sim 10^4$ cm^2/V sec would also be necessary.

Some preliminary experiments at temperatures around 20°K using semi-insulating GaAs with an electron mobility of 2×10^4 cm^2/Vsec at 77°K and a 10-kMc/sec ultrasonic wave are described elsewhere.[14]

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[14] J. R. A. Beale and M. Pomerantz (to be published).